Information and Maxwell's Refrigerator

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Joint work with Prof. Chris. Jarzynski and Dibyendu Mandal, submitted

Outline

- Introduction
- Information refrigerator
- Clausius's inequality
- Summary

Introduction

Is there any relation between information and heat?

Or is there any relation between the computer and the refrigerator?

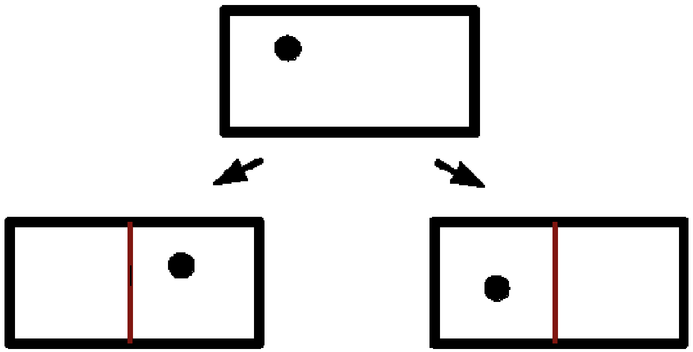
What is information?

What is one bit of information?

• Information is related to probability, once you know the probability distribution, you know the information amount from one measurement

• Obtaining information $\Longleftrightarrow$ Reducing uncertainty

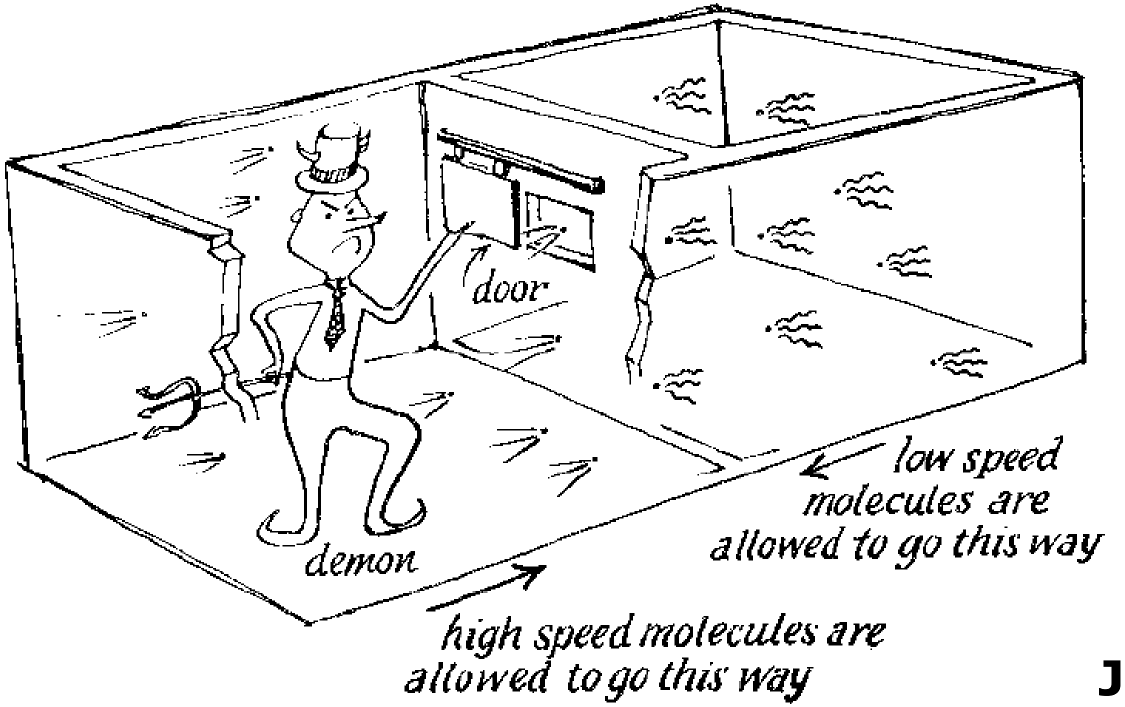
• Shannon Information amount: $$I = -\sum_i p_i \log_2 p_i$$



$p_R = \frac{1}{2}$ $p_L = \frac{1}{2}$

$$I = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} = 1 \text{ (bit)}$$

Maxwell's demon thought experiment



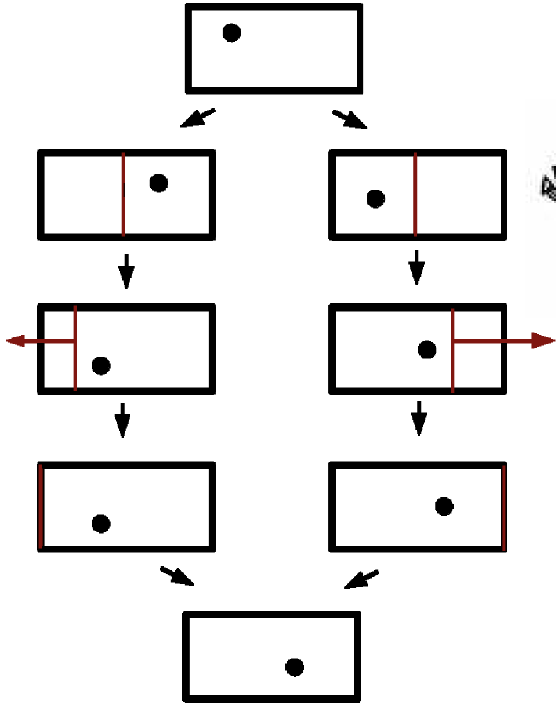
Theory of Heat (Longmans, London, 1871)

James Clerk Maxwell

1831 - 1879

Szilard's engine

Leo Szilard
1898-1964

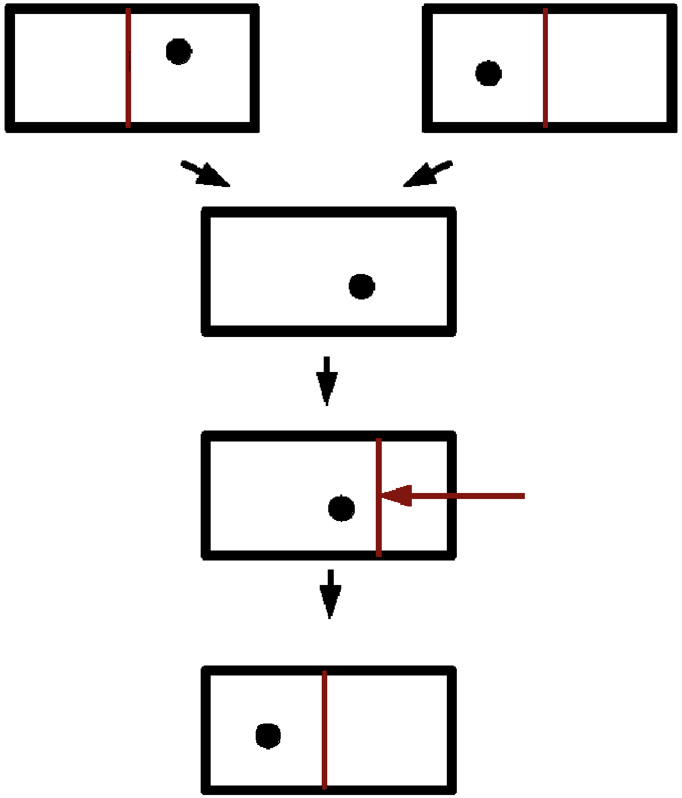


Szilard's Single Molecule
Engine (1929)

Landauer's principle (1961):

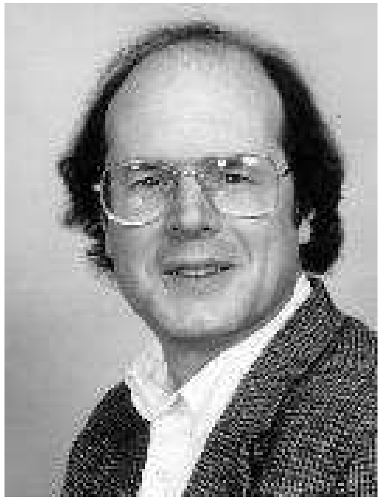
Each bit of lost information will lead to an release of amount of KTln 2 heat

Rolf Landauer
1927-1999

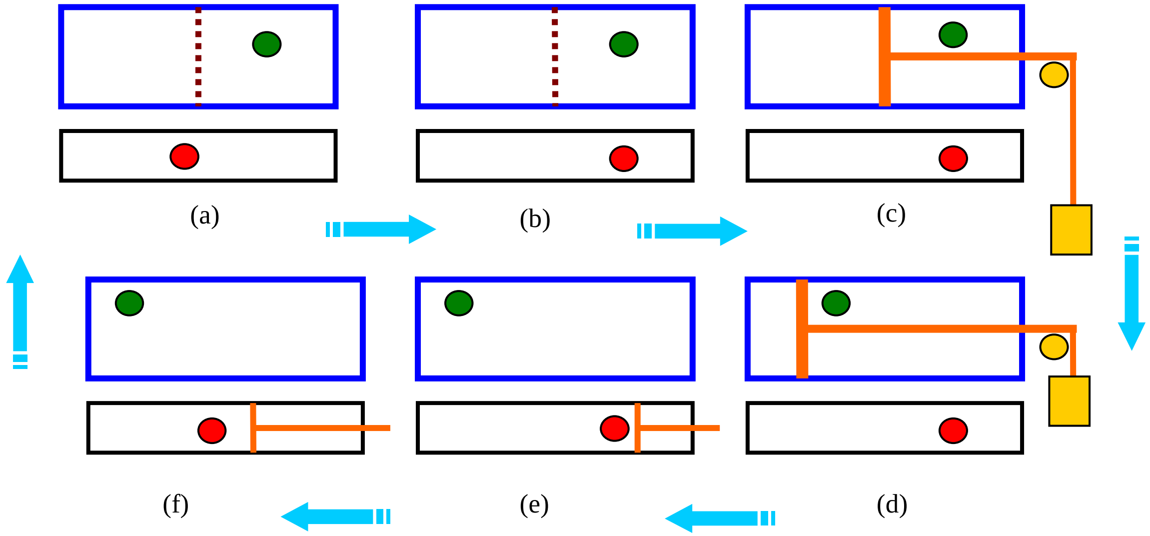


$$W = \int_{V}^{V/2} P dV = \int_{V}^{V/2} \frac{KT}{V} dV = -KT \ln 2$$

There is a lower bound of heat a computer must dissipate to process a given amount of information.



Charles H. Bennett
1943-



"The erasure of the memory of the demon compensates the entropy decreases and thus save the second law."

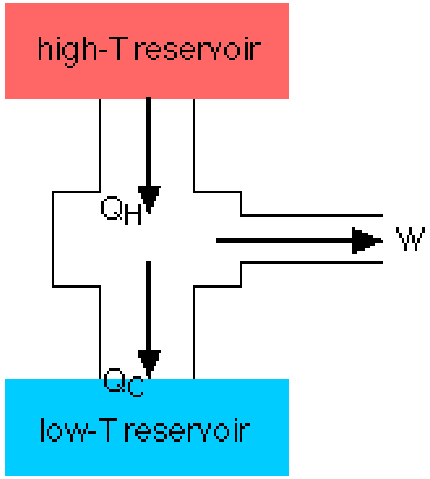
Is it possible to build an autonomous Maxwell's refrigerator without a demon?

- This refrigerator needs to rectify thermal fluctuations
- No intelligent creature (demon) is involved
- Achieving heat flow against temperature gradient by utilizing information
- The erasure of information compensates the entropy decrease

Information refrigerator

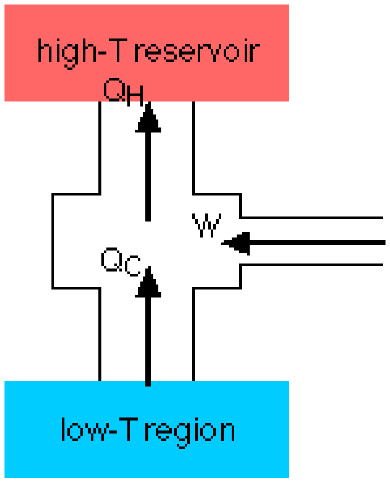
Heat engine and Refrigerator

Heat engine

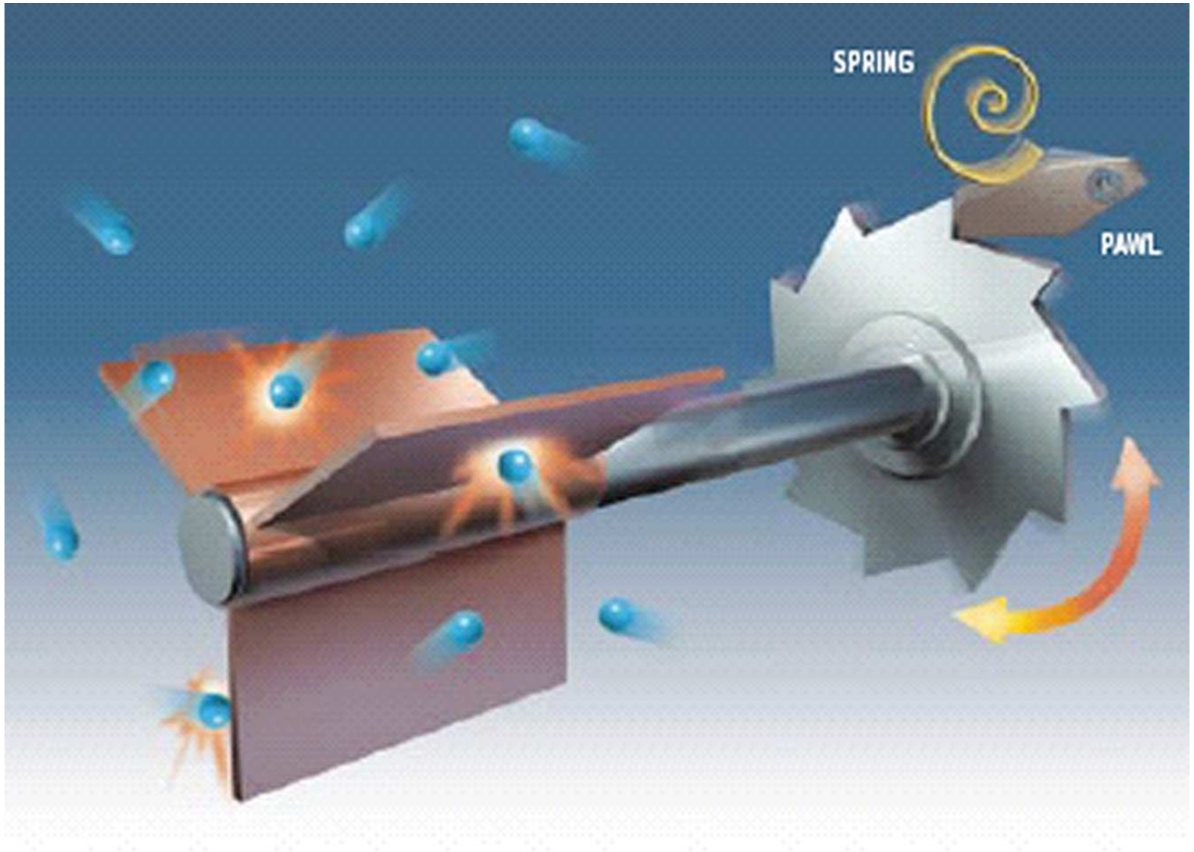


No thermal Fluctuation!

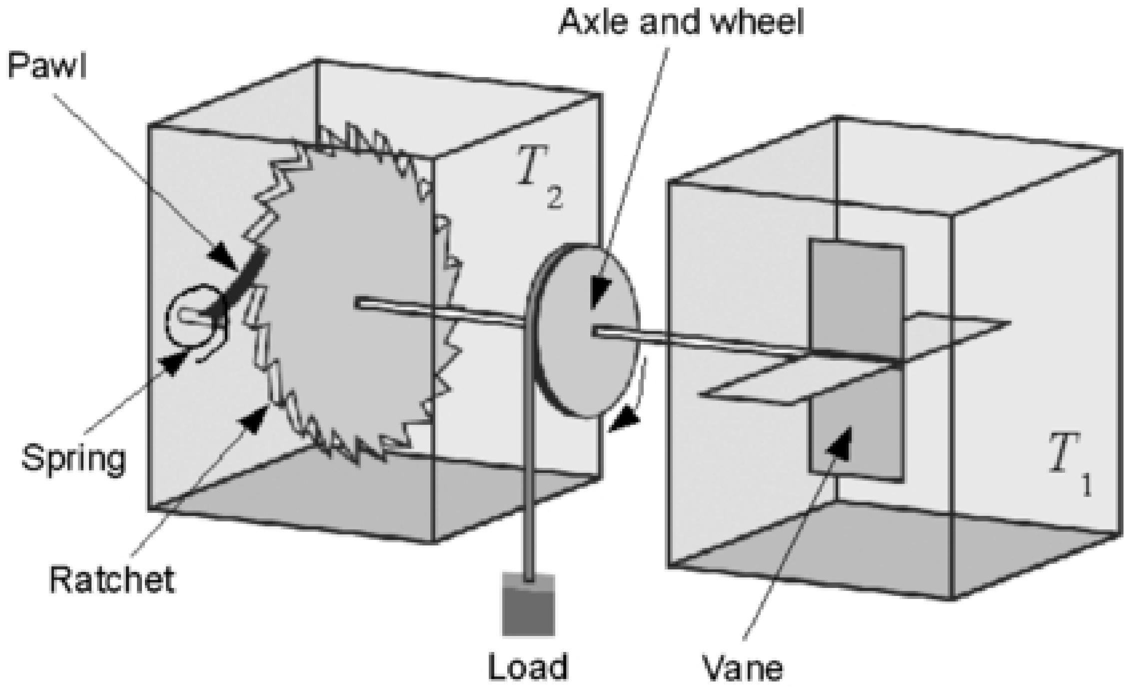
Refrigerator (Heat pump)



Fluctuations in small systems



Feynman's Ratchet and Pawl (1963)



Two competing Driving forces

•Temperature gradient

•Load mass

Heat Engine /Refrigerator

C. Jarzynski, et al PRE, 59, 6448 (1999); Z. C. Tu, J. Phys. A, 41, 312003 (2008)

- Rectifying thermal fluctuations, either a heat engine or a refrigerator
- No information is involved

In order to mimicking Maxwell's thought experiment, we need to input information instead of mechanical work

Feynman refrigerator and information refrigerator

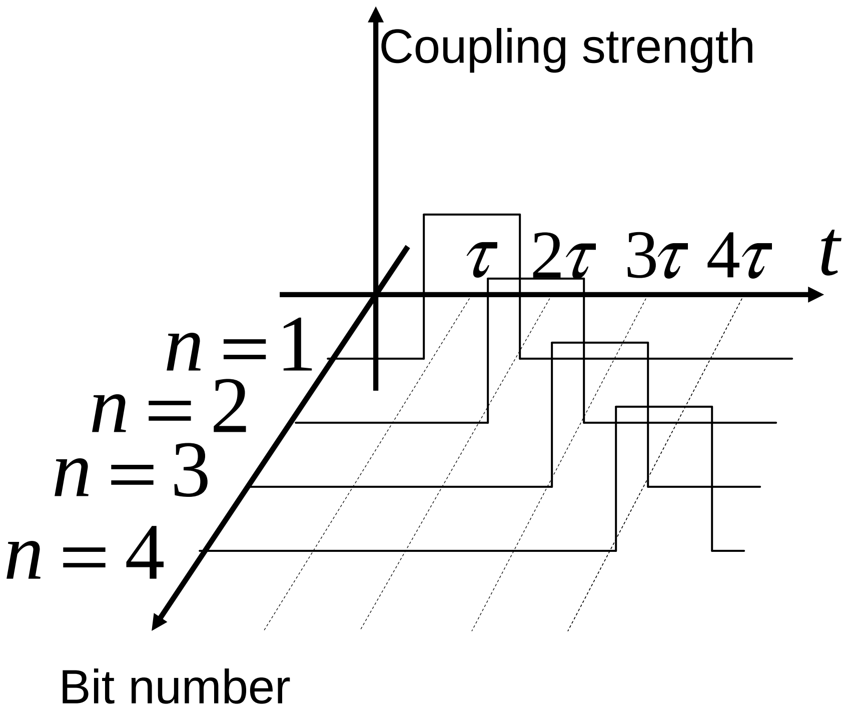
The Feynman refrigerator

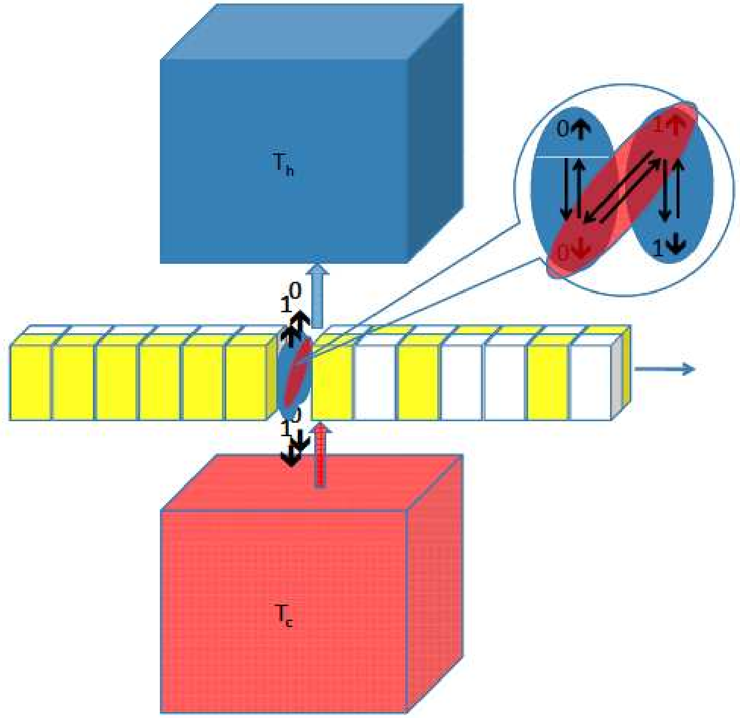
- This refrigerator needs to rectify thermal fluctuations
- Input mechanical work
- The conjugate cycle is a heat engine
- No information content is involved

The information refrigerator

- This refrigerator needs to rectify thermal fluctuations
- Input low entropy memory unit
- The conjugate is an information eraser
- No mechanical work is involved

Schematic figure of the information refrigerator





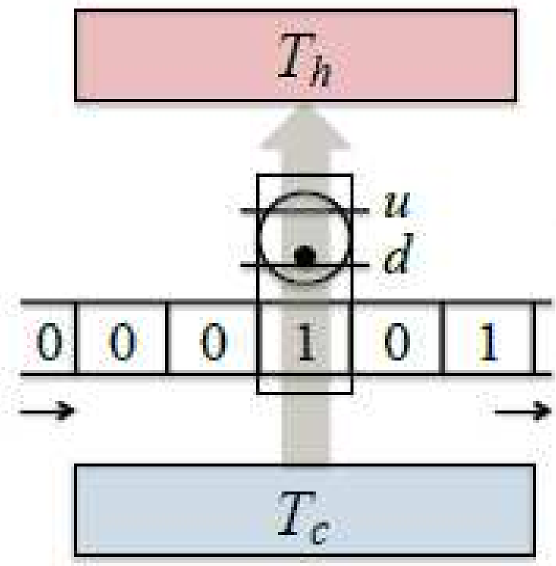
What is fixed?

Temperatures of the two reservoirs T_h T_c

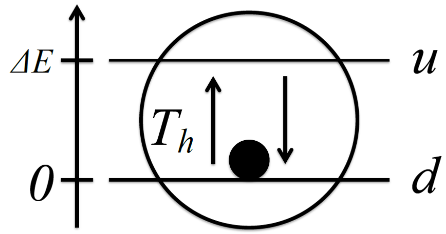
The initial probability distribution of the bits P_0^B P_1^B

The period of interaction between the two-level system and every bit τ

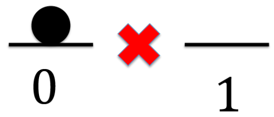
Information refrigerator



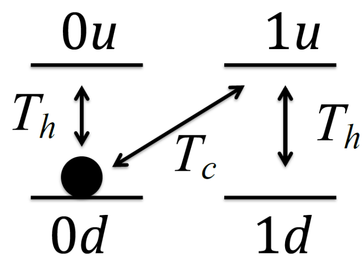
Two-level system
- Different energy
- Detailed balance



Input bits
- Equal energy
- No transition



Cooperative transition
- Heat exchange



Heuristic analysis: All bits prepared in “0”

Microscopic equations of motion

Classical master equation

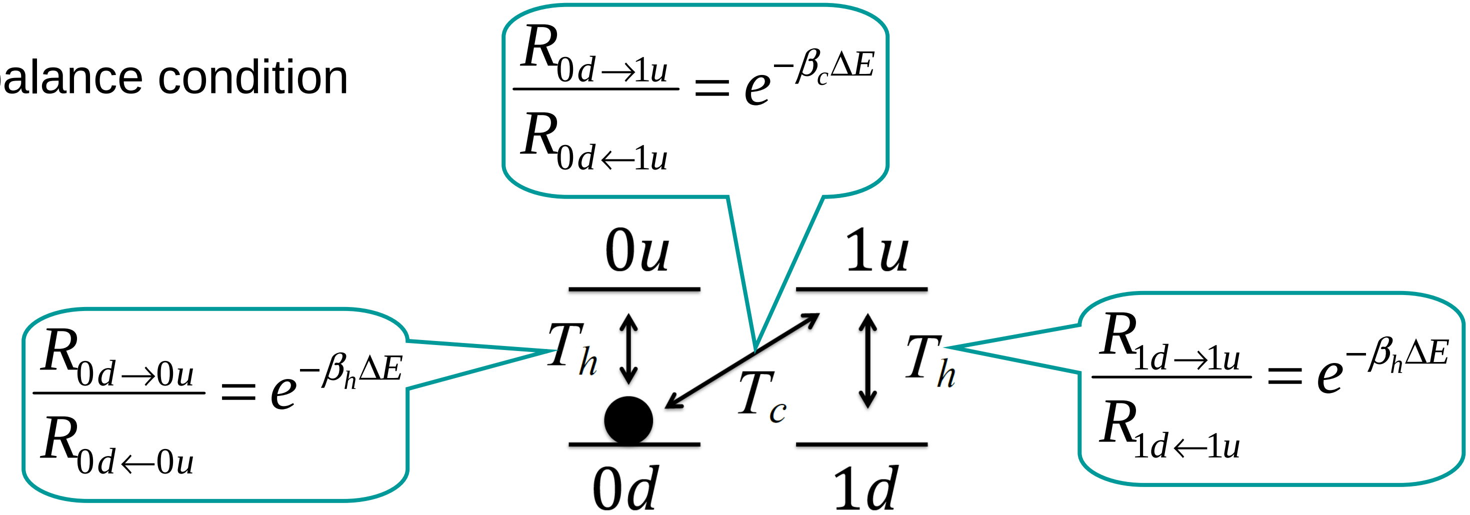
$$\frac{d\vec{P}(t)}{dt} = \Re \vec{P}(t) \qquad \vec{P}(t) = \begin{pmatrix} P_{0u}(t) \\ P_{0d}(t) \\ P_{1u}(t) \\ P_{1d}(t) \end{pmatrix}$$

Transition matrix

$$\mathcal{R} = \begin{pmatrix} \bullet & \gamma(1-\sigma) & 0 & 0 \\ \gamma(1+\sigma) & \bullet & 1+\omega & 0 \\ 0 & 1-\omega & \bullet & \gamma(1-\sigma) \\ 0 & 0 & \gamma(1+\sigma) & \bullet \end{pmatrix}$$

Microscopic equations of motion

Detailed balance condition



$$R_{0d\leftarrow 1u} = 1+\omega$$

$$R_{0d\to 1u} = 1-\omega$$

$$R_{d\to u} = \gamma(1-\sigma)$$

$$R_{d\leftarrow u} = \gamma(1+\sigma)$$

Strategy to solve the dynamics

Initial state of the two-level system and the bit

$$\begin{pmatrix} P_{0u}(0) \\ P_{0d}(0) \\ P_{1u}(0) \\ P_{1d}(0) \end{pmatrix} = \begin{pmatrix} P_0^{\ B} \times P_u^{\ D} \\ P_0^{\ B} \times P_d^{\ D} \\ P_1^{\ B} \times P_u^{\ D} \\ P_1^{\ B} \times P_u^{\ D} \end{pmatrix} \Rightarrow \begin{pmatrix} P_{0u}(\tau) \\ P_{0d}(\tau) \\ P_{1u}(\tau) \\ P_{1d}(\tau) \end{pmatrix} = e^{\Re \tau} \begin{pmatrix} P_0^{\ B} \times P_u^{\ D} \\ P_0^{\ B} \times P_d^{\ D} \\ P_1^{\ B} \times P_u^{\ D} \\ P_1^{\ B} \times P_u^{\ D} \end{pmatrix}$$

The marginal distribution of the two-level system

$$\begin{pmatrix} P_u^{\ D}(\tau) \\ P_d^{\ D}(\tau) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_{0u}(\tau) \\ P_{0d}(\tau) \\ P_{1u}(\tau) \\ P_{1d}(\tau) \end{pmatrix}$$

Periodic steady state of the two-level system

$$\begin{pmatrix} P_u^{D,ps}(\tau) \\ P_d^{D,ps}(\tau) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_{0u}(\tau) \\ P_{0d}(\tau) \\ P_{1u}(\tau) \\ P_{1d}(\tau) \end{pmatrix} = \begin{pmatrix} P_u^{D,ps}(0) \\ P_d^{D,ps}(0) \end{pmatrix}$$

The probability distribution of the outgoing bits

$$\begin{pmatrix} P_0^{B}(\tau) \\ P_1^{B}(\tau) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} e^{\Re\tau} \begin{pmatrix} P_0^B(0) & 0 \\ 0 & P_0^B(0) \\ P_1^B(0) & 0 \\ 0 & P_1^B(0) \end{pmatrix} \begin{pmatrix} P_u^{D,ps}(0) \\ P_d^{D,ps}(0) \end{pmatrix}$$

The periodic steady sate depends on the period τ

Two competing driving forces

Temperature gradient

$$\varepsilon = \frac{\omega - \sigma}{1 - \omega\sigma} = \tanh \frac{(\beta_c - \beta_h)\Delta E}{2}$$

Low entropy of the input bits

$$\delta = P_0^B(0) - P_1^B(0)$$

If the former wins $\varepsilon > \delta$ It is an information erasure

If the later wins $\varepsilon < \delta$ It is an information refrigerator

Heat flux from the master equation

$$Q_{c\to h} = \left[P_1^B(\tau) - P_1^B(0)\right]\Delta E$$

Where

$$P_1^B(\tau) - P_1^B(0) = \frac{\delta - \varepsilon}{2}\eta(\Lambda)$$

$$\eta(\Lambda) = \frac{\nu_2 P + \nu_3 Q}{P + Q},$$

$$P = \mu_2\left(\mu_4\nu_3 + \mu_1\nu_1\right) \quad , \quad Q = \mu_3\left(\mu_4\nu_2 + \mu_1\nu_1\right),$$

$$\nu_1 = 1 - e^{-2\gamma\,\tau} \quad , \quad \mu_1 = (\delta + \sigma)\,\omega,$$

$$\nu_2 = 1 - e^{-(1+\gamma-\alpha)\,\tau} \quad , \quad \mu_2 = \alpha + \gamma + \sigma\,\omega,$$

$$\nu_3 = 1 - e^{-(1+\gamma+\alpha)\,\tau} \quad , \quad \mu_3 = \alpha - \gamma - \sigma\,\omega,$$

$$\alpha = \sqrt{1 + \gamma^2 + 2\gamma\sigma\omega} \quad , \quad \mu_4 = 1 - \delta\,\omega.$$

Heat flow from low temperature to high temperature

$$Q_{c\rightarrow h} = \Delta E \frac{\delta - \varepsilon}{2} \eta(\Lambda)$$

Shannon entropy change of every bit

$$\Delta S_B = -k_B \left[P_0^B(\tau) \ln P_0^B(\tau) + P_1^B(\tau) \ln P_1^B(\tau) \right] + k_B \left[P_0^B(0) \ln P_0^B(0) + P_1^B(0) \ln P_1^B(0) \right]$$

"Phase diagram" of the device

Information
Refrigerator

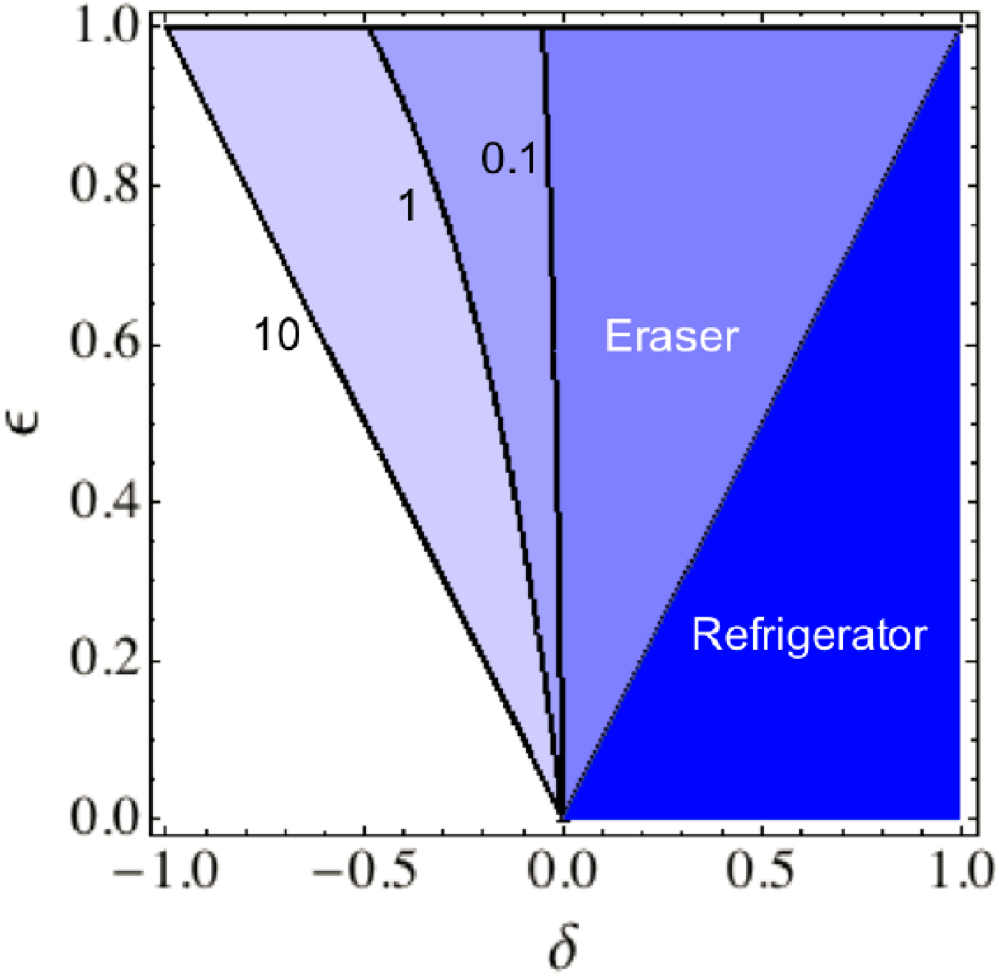
$$Q_{c\to h} > 0$$

$$\Delta S_B > 0$$

Information
Eraser

$$Q_{c\to h} < 0$$

$$\Delta S_B < 0$$



Clausius inequality

$$Q_{c\to h}(\beta_h - \beta_c) + \Delta S_B \geq 0$$

Information entropy increase in every bit

$$\Delta S_B = -k_B\left[P_0^B(\tau)\ln P_0^B(\tau) + P_1^B(\tau)\ln P_1^B(\tau)\right]$$
$$+ k_B\left[P_0^B(0)\ln P_0^B(0) + P_1^B(0)\ln P_1^B(0)\right]$$

The second law is not violated if we identify the information entropy with the thermodynamic entropy

Lower bound on heat dissipation in information processing

$10^{-21} J$ Heat dissipated when one bit is erased

In modern silicon device it is 1000 times higher than that limit

Antoine Berut et al., Nature, 483, 187 (2012)

Experimental relevance

Colloid particle
- G. M. Wang et al, Phys. Rev. Lett., 89, 050601 (2002)
- V. Blickle et al, Phys. Rev. Lett., 96, 070603 (2006)
- Tongcang Li et al, Science, 328, 1673 (2010)

Biosystems
Jan liphardt et al Science, 296, 1832 (2002)

Superconducting qubit
H. T. Quan et al Rev. Lett. 97, 180402 (2006)

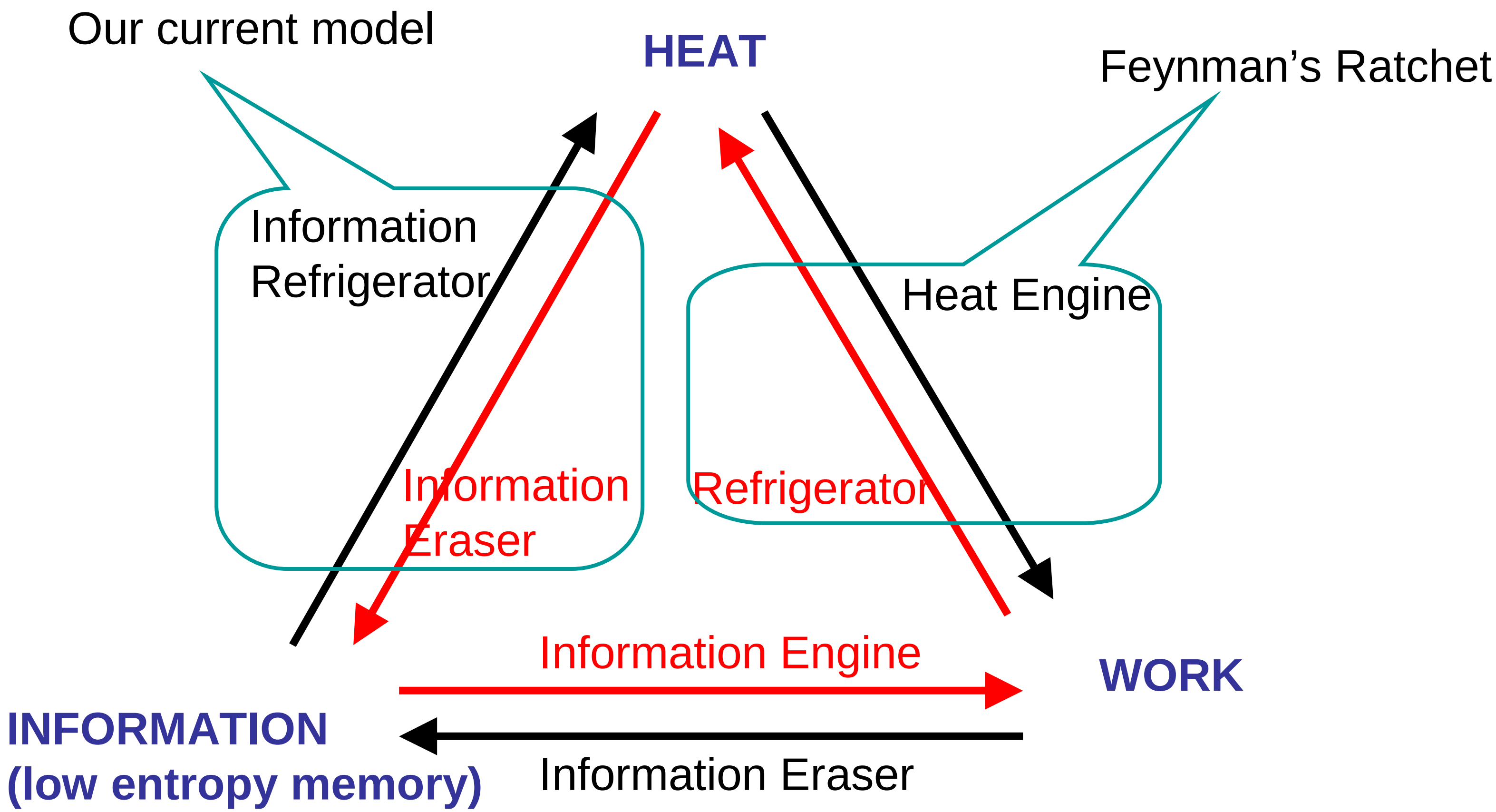
Trapped ion system
G. Huber et al Phys. Rev. Lett. 101, 070403 (2008)

Reminding

"Heat can never pass from a colder to a warmer body without some other change connected therewith, occurring at the same time." --- R. Clausius, (1854)

~~对于任意循环运转装置，在不输入功的情况下，而自发性地产生使热由冷体传向热体之效应是不可能的。---维基百科网站，（2013）~~

Summary



Driving forces: (1) temperature gradient, (2) mechanical work, (3) low entropy memory

Summary

•We introduce an autonomous model to generate heat flow against the thermal gradient mimicking the Maxwell’s original idea

•Heat can be pumped from a low temperature to a high temperature reservoir if there is a memory register to which we can write information

•The increases of the information entropy of the memory compensates the decreases in the thermodynamic entropy.

Thank you!